Pulse Width Modulation - fundamentals

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Outline

01 Fundamentals of inverters

02 Inverter voltages

03 Pulse Width Modulation

04 Harmonic Spectrum

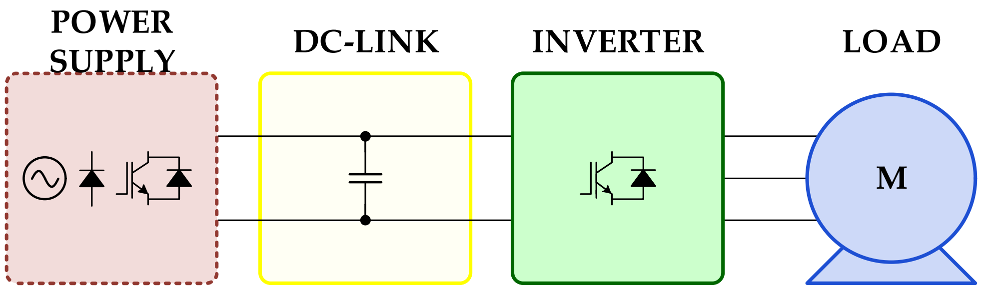
05 Limitations on PWM parameters

06 Regular and natural sampling methods and saw-tooth carrier signal

07 Comparison of harmonic spectra

Voltage source inverter

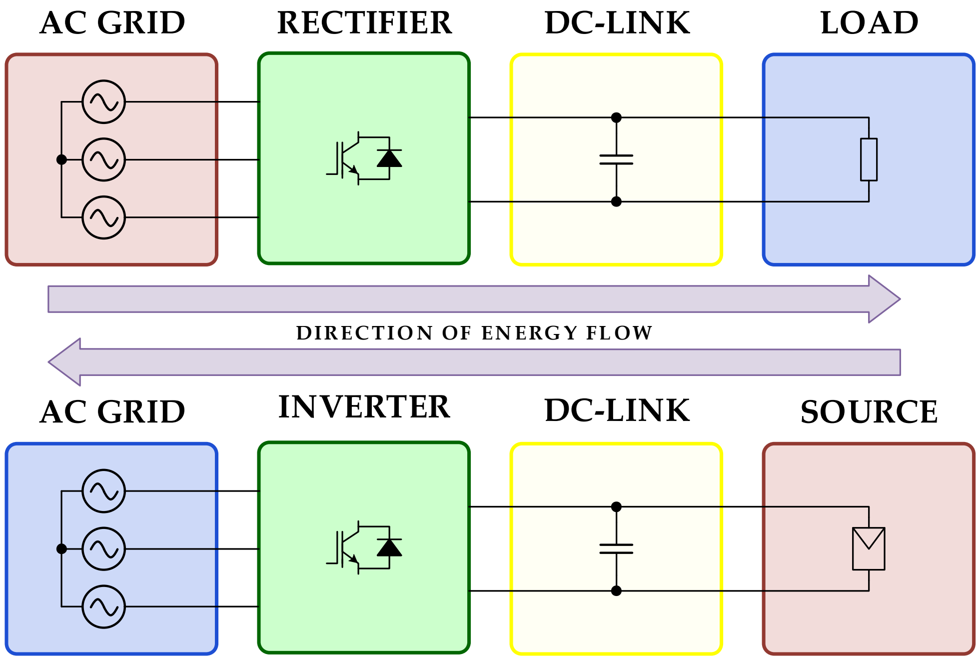
Voltage-source inverter is a converter that transforms a dc-link voltage into an ac voltage



In a voltage-source inverter, the dc-link is a capacitor, while the load exhibits a resistive–inductive character

There also exist current-source inverters, which are used less frequently and include an inductor in the dc circuit. In this case, the load has a resistive–capacitive character

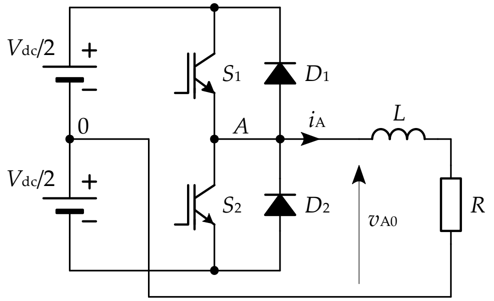
Depending on the direction of the power flow, the same converter can operate either as an **inverter** or as a transistor-based **rectifier** (*PWM rectifier, active front-end*).



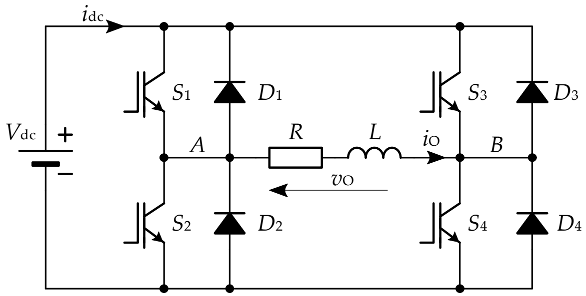
Topology of 2-level converter

The most commonly used inverter is the **two-level converter**. It consists of switching legs composed of two transistors with antiparallel diodes.

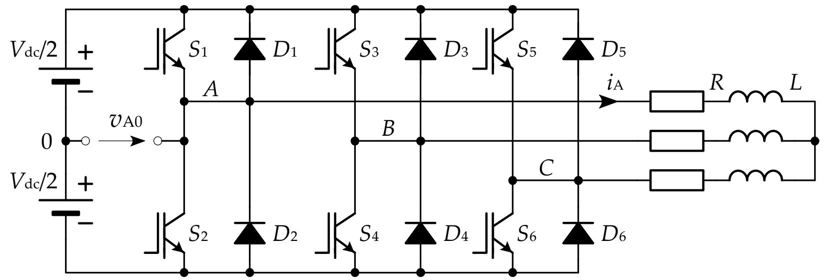
Single-phase half-bridge converter



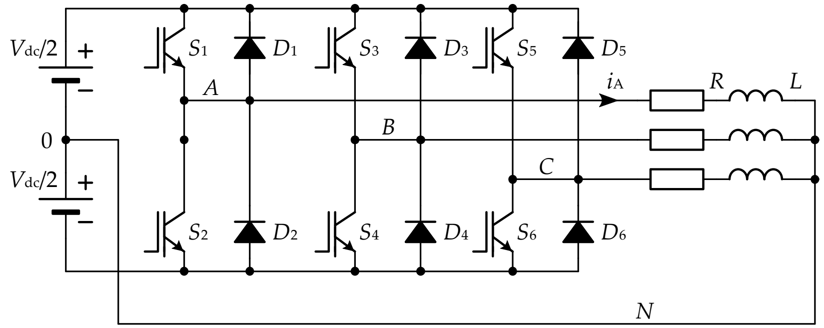
Single-phase full-bridge converter



Three-phase, three-leg converter

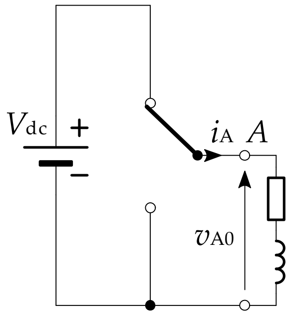
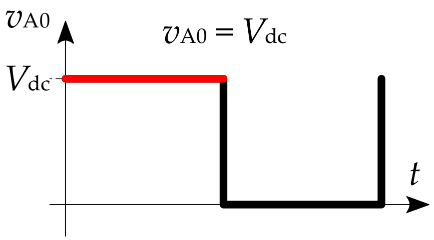
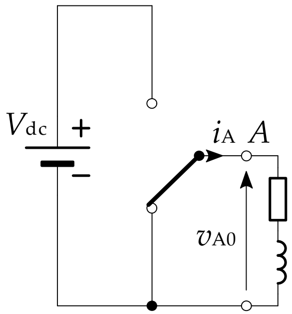
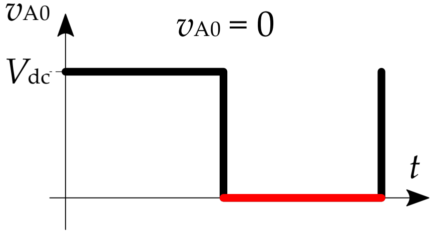


Three-phase, three-leg, four-wire converter

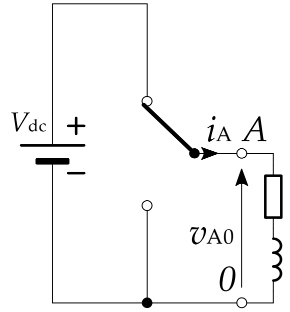


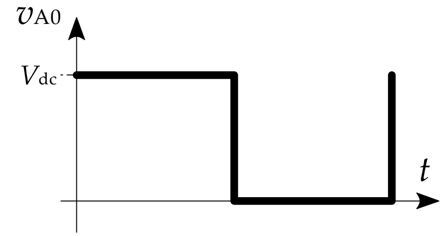
2-level converter – voltage levels

A transistor leg can be represented as a switch

Switch position	Schematic	Output voltage
The output is connected to the positive terminal of the DC voltage source		2 voltage levels — hence the name of the converter 
The output is connected to the negative terminal of the DC voltage source		

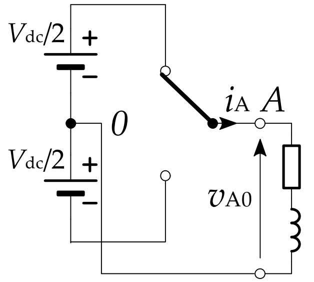
The negative terminal of the load is connected to the negative terminal of the dc-link.

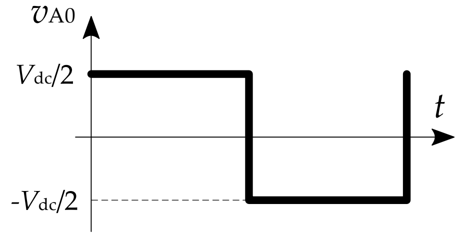




This is not an AC voltage (a DC component is present).
It is a DC–DC converter

The negative terminal of the load is connected to the midpoint of the dc-link.





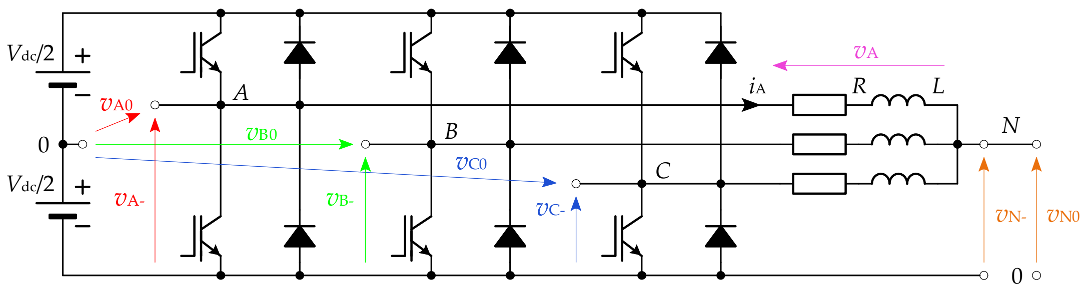
This is an AC voltage

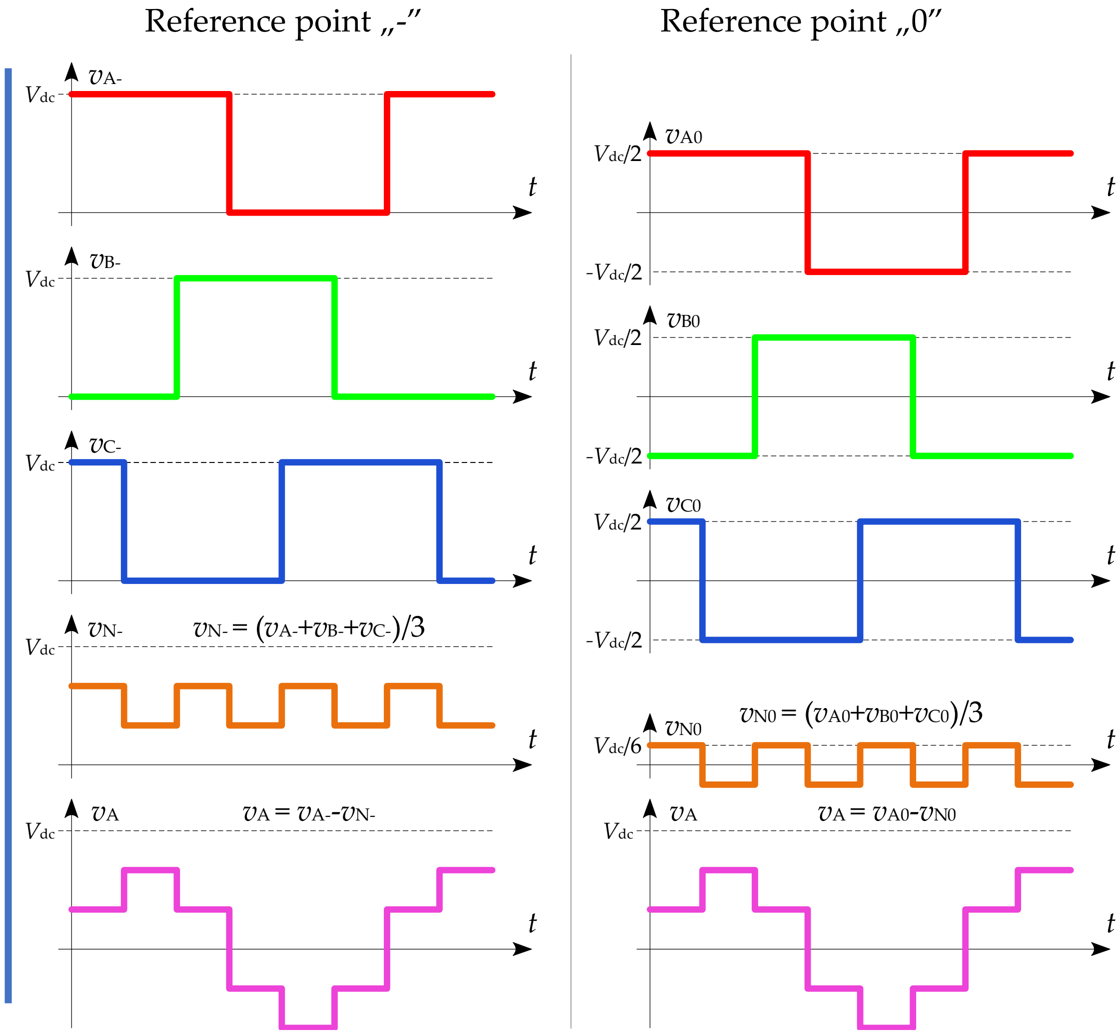
Virtual midpoint of the dc-link

In many converters, when one or several parallel-connected capacitors are used, the DC-link midpoint does not exist. This applies to the converter:

- Single-phase, full-bridge converter
- Three-phase, three-leg converter

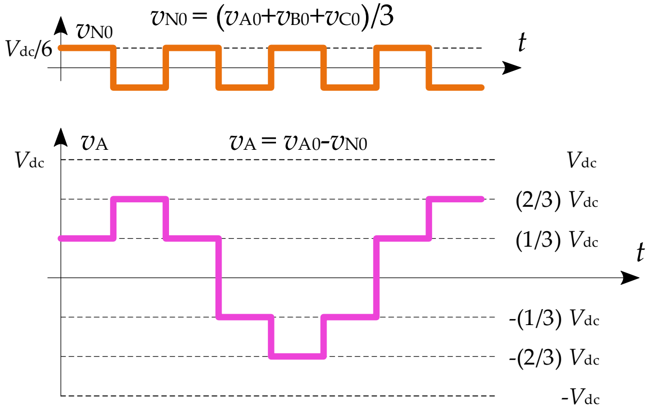
Defining such a point may facilitate the analysis of the converter's operation, but it is not necessary.



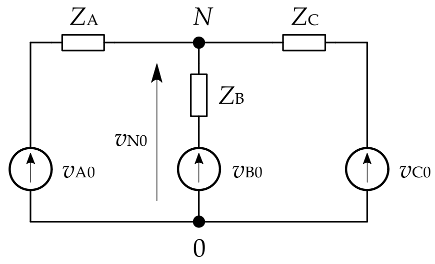


Star point voltage

The star-point voltage is also referred to as the common-mode voltage. It is not equal to zero because the sum $v_{A0}+v_{B0}+v_{C0} \neq 0$.



The expression for v_{N0} can be derived using the nodal analysis for a symmetric three-phase load supplied by the voltages v_{A0}, v_{B0} i v_{C0}.



For symmetric three-phase load $Z_A = Z_B = Z_C = Z$.

From nodal analysis the star-point voltage v_{N0}.

$$\left(\frac{1}{Z_A}+\frac{1}{Z_B}+\frac{1}{Z_C}\right)v_{N0}=\frac{v_{A0}}{Z_A}+\frac{v_{B0}}{Z_B}+\frac{v_{C0}}{Z_C} \quad (1)$$

$$\frac{3}{Z}v_{N0}=\frac{1}{Z}\left(v_{A0}+v_{B0}+v_{C0}\right) \quad (2)$$

$$v_{N0}=\frac{v_{A0}+v_{B0}+v_{C0}}{3} \quad (3)$$

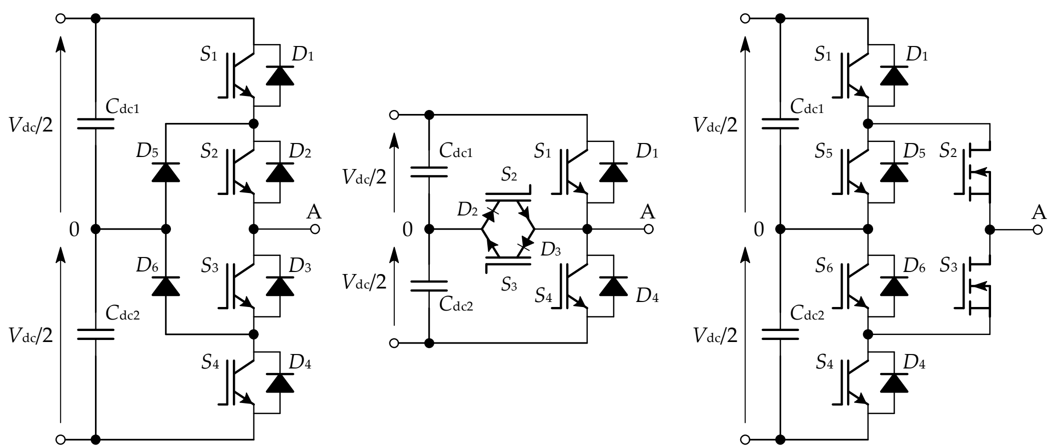
Eq. (3) is valid only for the symmetric three-phase loads, but such loads are most often connected to inverters.

Multilevel converters

There are converters with a higher number of levels.

Three-level converters:

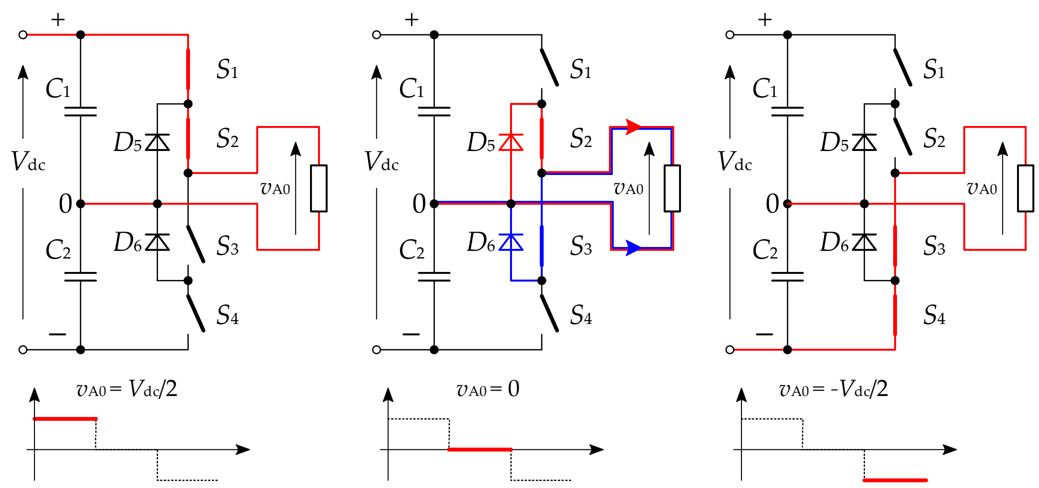
NPC	NPC-T	ANPC
Neutral Point Clamped	*Neutral Point Clamped Type T*	*Advanced Neutral Point Clamped*



Three-level converters use semiconductor switches with lower blocking voltages and make it possible to:

- reduce power losses
- increase the dc-link voltage.

Operation of the NPC converter



v_{A0}	Switching states			
	S_1	S_2	S_3	S_4
$V_{dc}/2$	**1**	**1**	**0**	**0**
0	**0**	**1**	**1**	**0**
$-V_{dc}/2$	**0**	**0**	**1**	**1**

Complementary pairs:

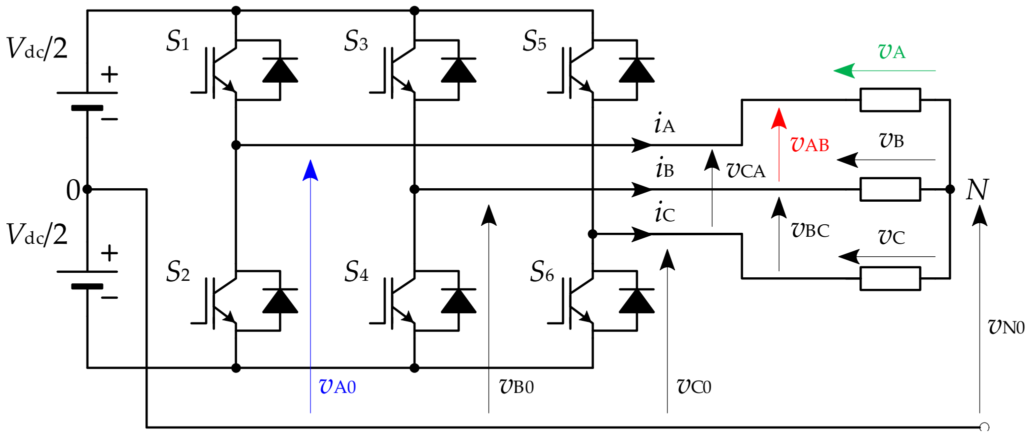
$S_1 = \neg S_3,\ S_2 = \neg S_4$

Inverter voltage levels

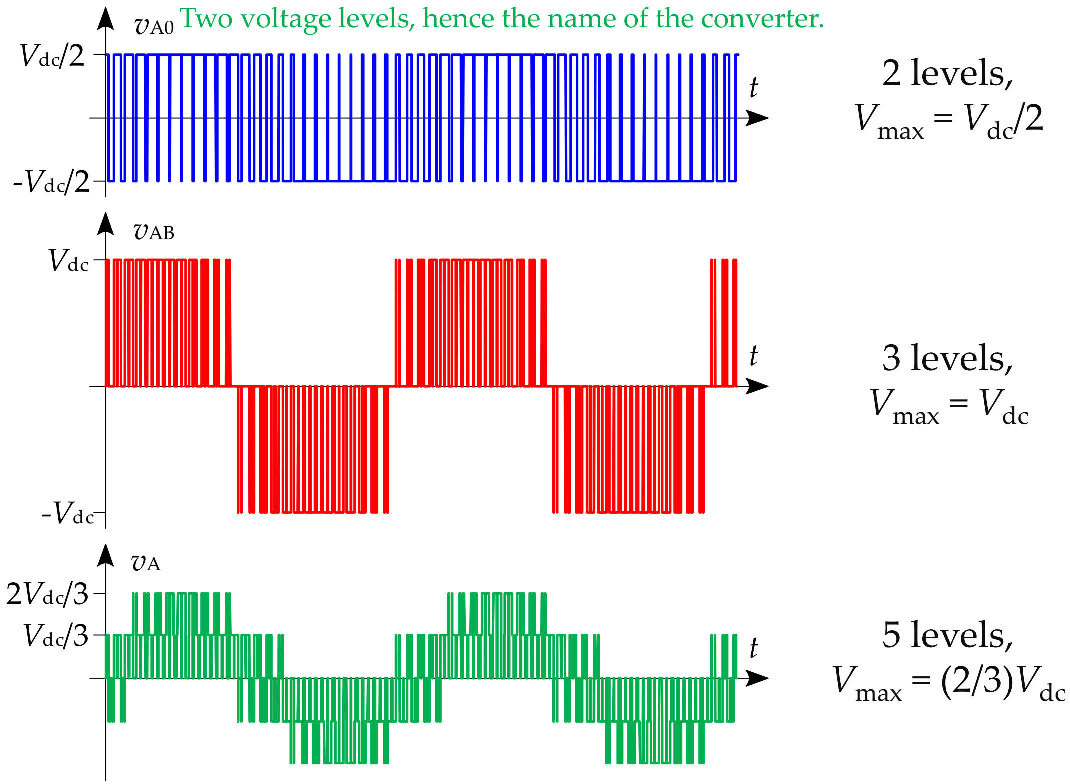
In a three-phase converter, the following voltages are distinguished:

- output voltages v_{A0}, v_{B0} and v_{C0}
- line-to-line voltages v_{AB}, v_{BC} and v_{CA}
- phase voltages v_A, v_B and v_C.

Each of these voltages contains a different number of levels.

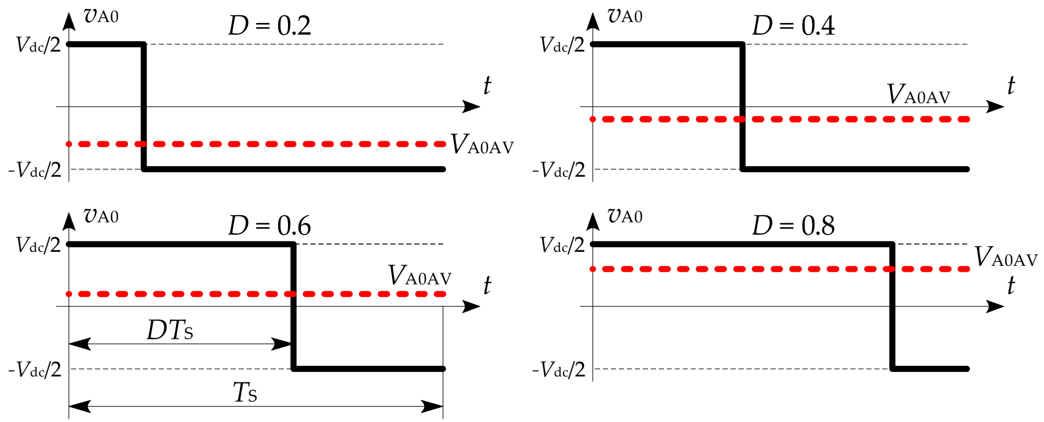


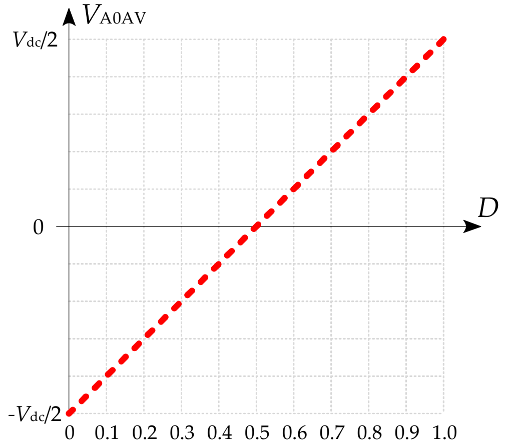
When pulse-width modulation (PWM) is applied, the voltages are switched.



Pulse width modulation - fundamentals

Output voltage waveforms for different duty cycles D

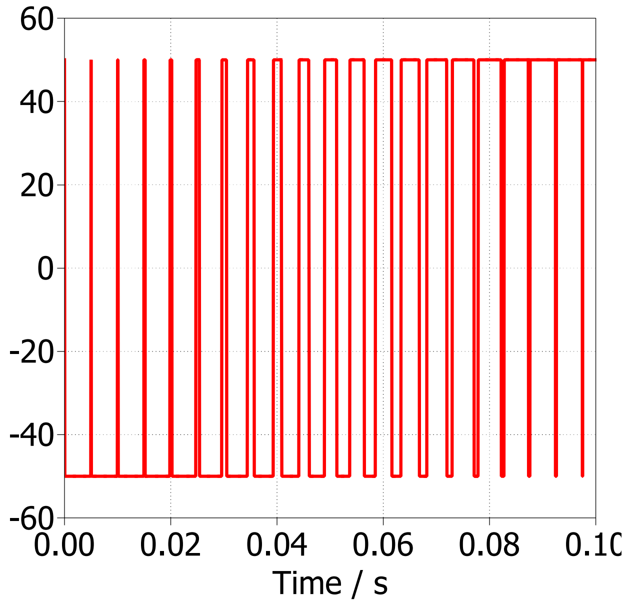




Duty cycle D changes the average value of the output voltage

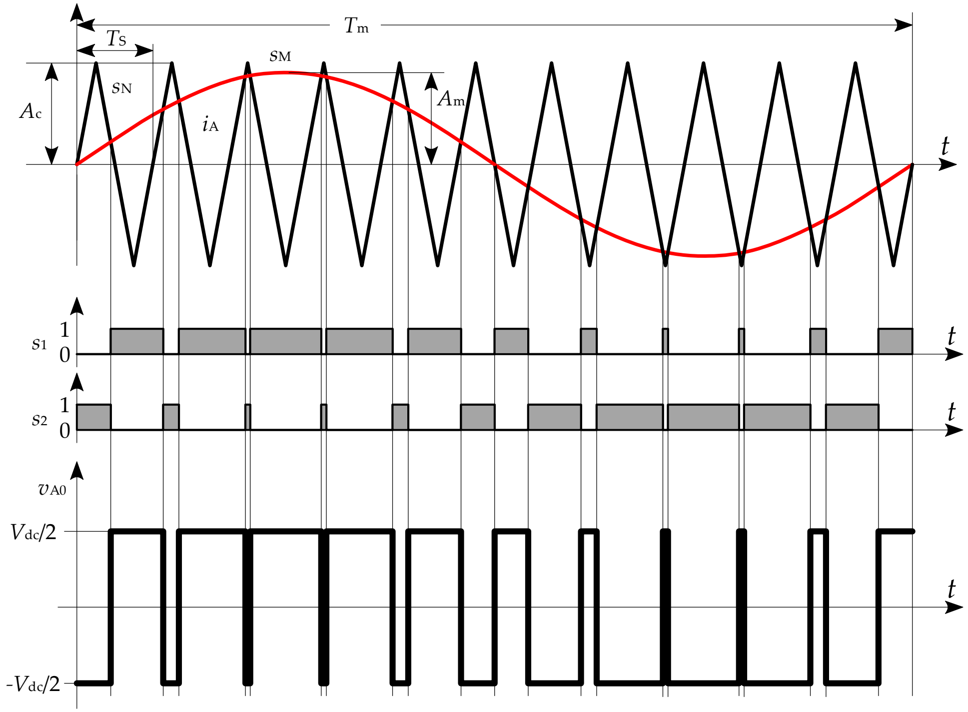
$$V_{A0AV} = V_{dc} D - \frac{V_{dc}}{2} \qquad (4)$$

The idea of pulse-width modulation is to vary the duty cycle D in such a way that the average value (averaged over the switching period T_S) reproduces the signal that controls the duty-cycle variation.



Pulse width modulation - fundamentals

In inverters, instead of using the concept of the duty cycle D in pulse-width modulation, the term modulating signal s_M is used.



The switching signals of the converter transistors (s_1, s_2) are obtained by comparing the modulating signal with a triangular carrier signal s_n of frequency $f_S = 1/T_S$.

Upper switch S_1 turns on when signal s_1

$$s_1 = 1 \text{ when } \quad s_M > s_N$$

Lower switch S_2 turns on when signal s_2

$$s_2 = 1 \text{ when } \quad s_M < s_N$$

When the modulating signal s_M is a sinusoid of the form of $s_M = A_m \sin(2\pi f_m t)$, two parameters are defined:

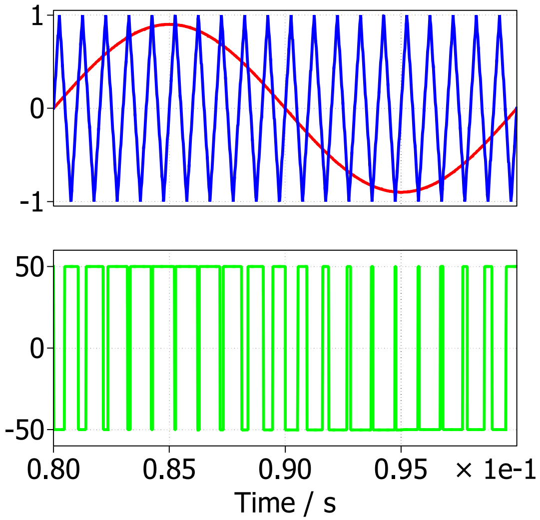
Modulation index $$m_a = \frac{A_m}{A_c}$$

Frequency index $$m_f = \frac{f_S}{f_m} = \frac{T_m}{T_S}$$

Pulse width modulation - fundamentals

Examples:

Different modulation indices $m_a = 0.9$ and $m_a = 0.4$.

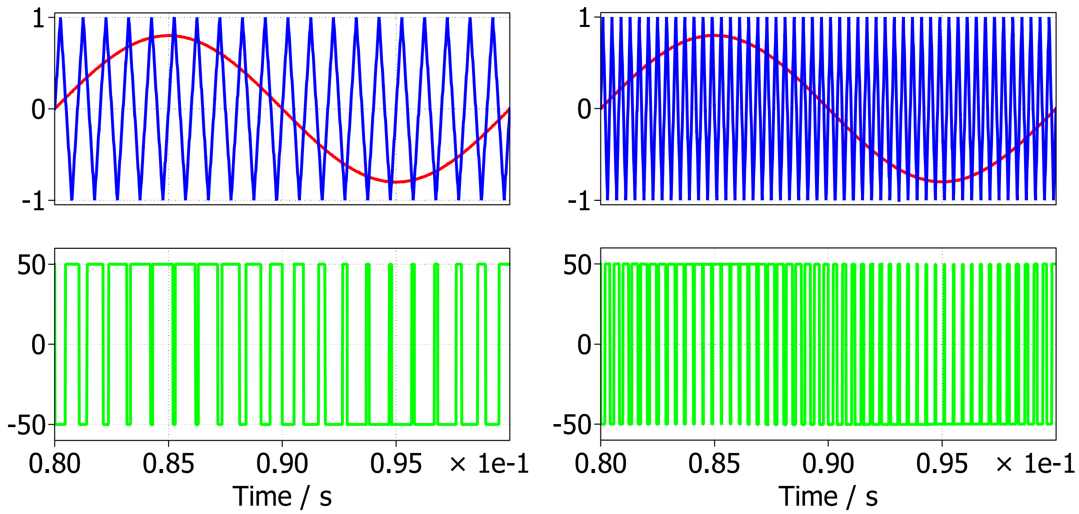


Other parameters unchanged:

$f_m = 50$ Hz, $m_f = 20$, $V_{dc} = 100$ V.

Examples:

Different frequency indices $m_f = 20$ and $m_f = 50$.



Other parameters unchanged:

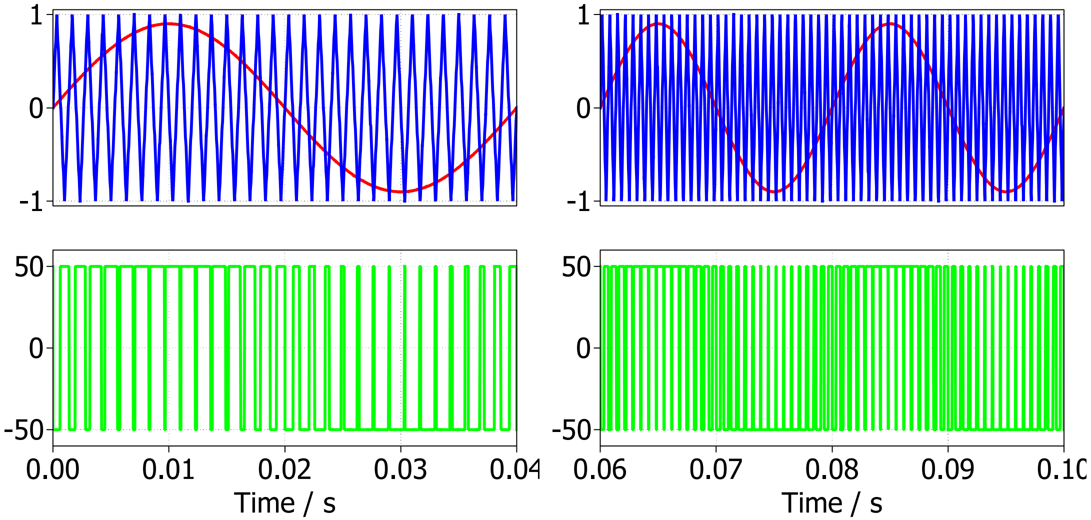
$f_m = 50$ Hz, $m_a = 0.8$, $V_{dc} = 100$ V.

Pulse width modulation - fundamentals

Examples:

Different values of fundamental frequency

f_m = 25 Hz and f_m = 50 Hz.



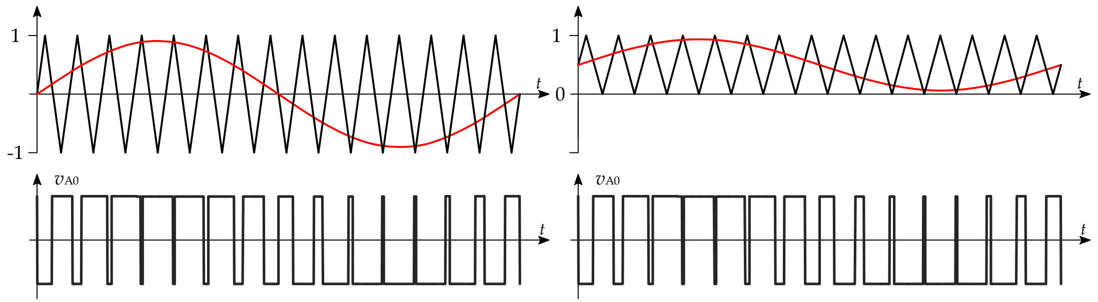
Other parameters unchanged:

$m_a = 0.9$; $m_f = 30$ (the switching frequency is changed due to a change of f_m); $V_{dc} = 100$ V.

The carrier signal s_N may vary in the range from –1 to 1 or from 0 to 1. In microcontroller implementations, the carrier signal corresponds to the value of a counter that increments from 0 to an integer value of several hundred. Examples of using a carrier signal varying in the ranges (–1, 1) and (0, 1).

The modulating signal s_M must be adjusted accordingly

$$s_M = m_a \sin(2\pi f_m t) \qquad s_M = \frac{1}{2}\left(1 + m_a \sin(2\pi f_m t)\right)$$

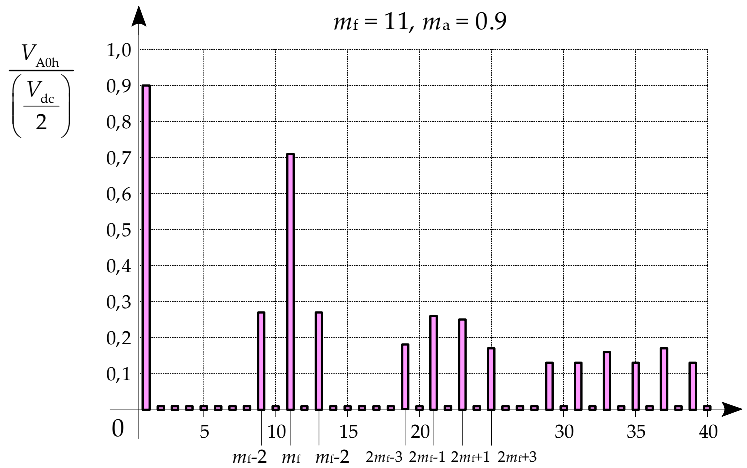


Regardless of the chosen method, the output voltage is the same in both cases.

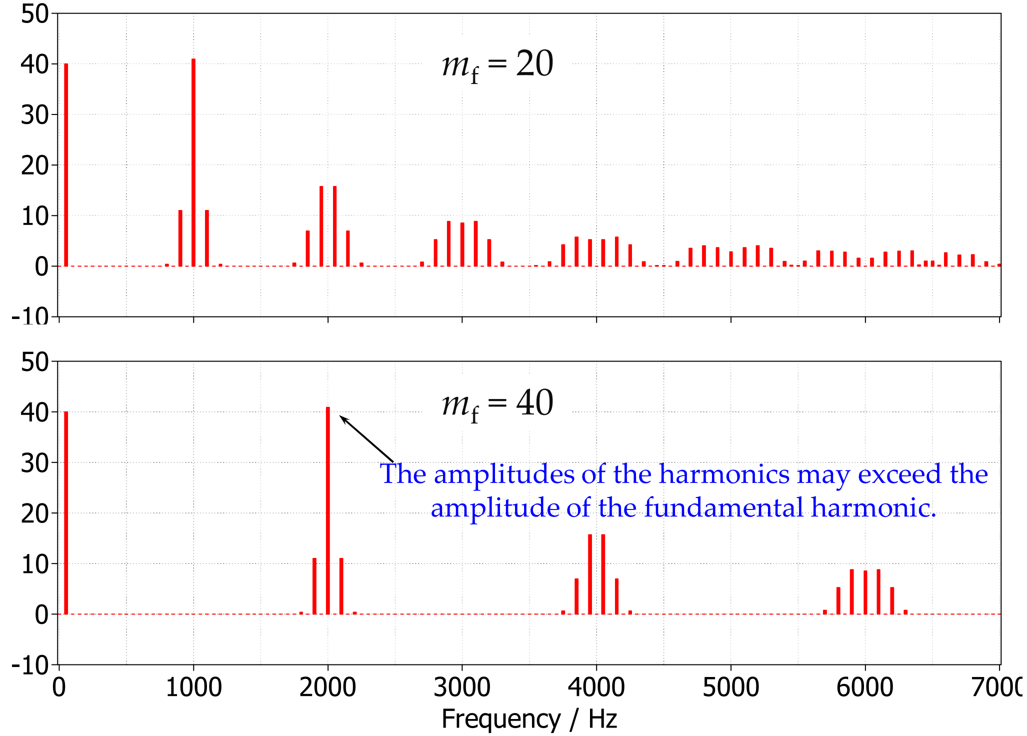
Pulse width modulation – harmonic spectrum

Increasing the switching frequency improves the shape of the output current waveform, but it also increases the switching losses in the inverter.

In the output voltage waveform, in addition to the fundamental harmonic, there are harmonics associated with the switching frequency and their higher-order components. Their orders depend only on m_a.

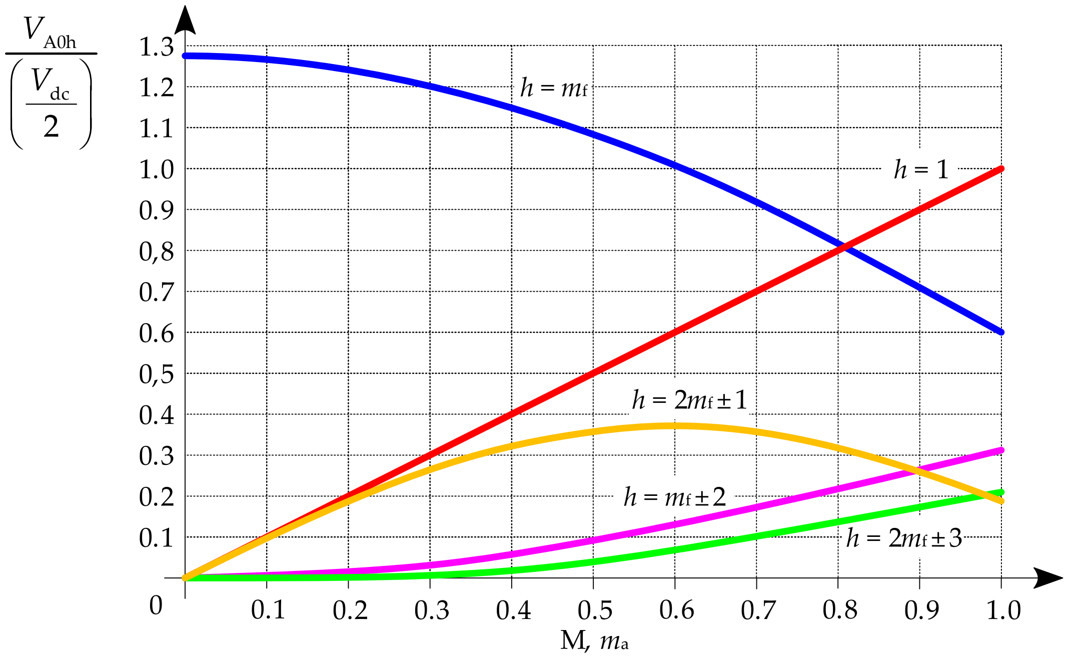


Simulation results in PLECS for $m_a = 0.8$.



The harmonic amplitudes do not change for the same modulation index m_a.

Pulse width modulation – harmonic spectrum



$h = 1$ – fundamental harmonic

$h = m_f$ – first harmonic of the carrier signal

$h = m_f \pm 2$ – sideband harmonics of the first carrier harmonic

$h = 2m_f \pm 1$ – sideband harmonics of the second carrier harmonic

$h = 2m_f \pm 3$ – sideband harmonics of the second carrier harmonic

Amplitudes of the most significant harmonics for different modulation indices m_a.

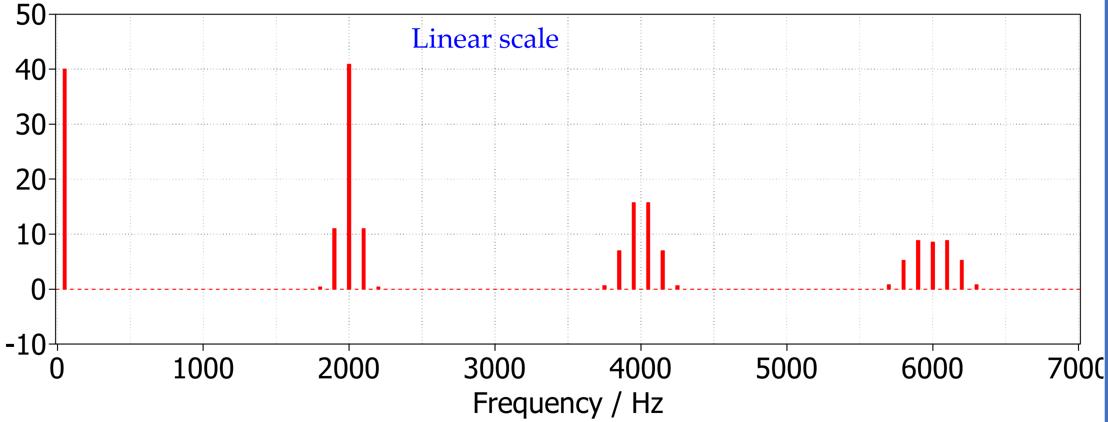
m_a	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
$h = 1$	1.00	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10
$h = m_f$	0.60	0.71	0.82	0.92	1.01	1.08	1.15	1.20	1.24	1.27
$h = m_f \pm 2$	0.32	0.27	0.22	0.17	0.13	0.09	0.06	0.03	0.02	0.00
$h = 2m_f \pm 1$	0.18	0.26	0.31	0.35	0.37	0.36	0.33	0.27	0.19	0.10
$h = 2m_f \pm 3$	0.21	0.18	0.14	0.10	0.07	0.04	0.02	0.01	0.00	0.00

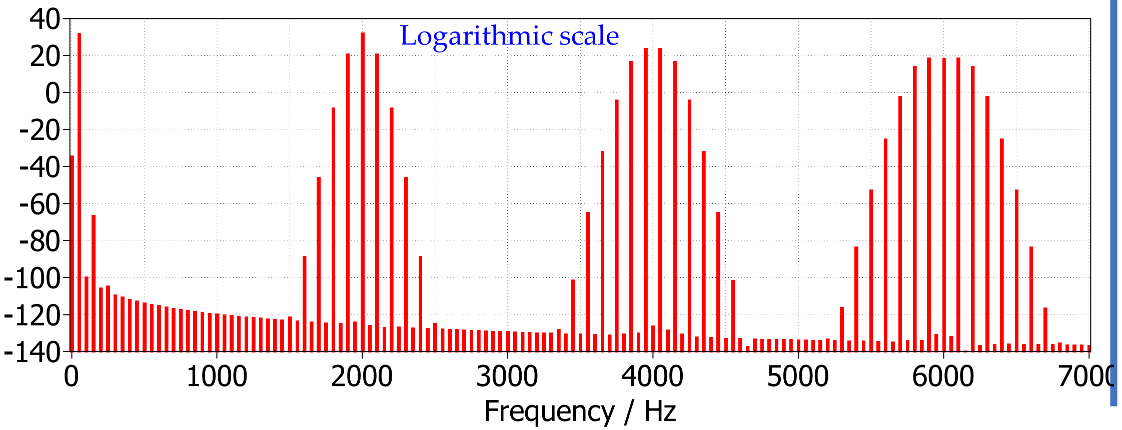
The linear relationship of the fundamental harmonic ($h = 1$) is a fundamental feature of modulation methods.

Apart from the harmonics listed above, additional sideband harmonics exist whose amplitudes are very small on a linear scale but can be revealed on a logarithmic scale (in dB).

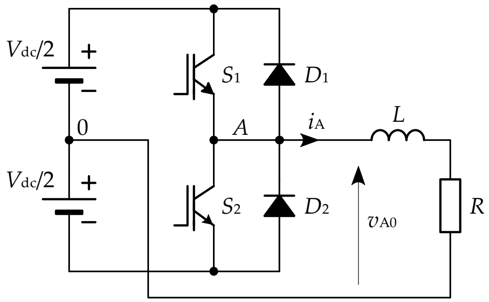
Pulse width modulation – harmonic spectrum

Simulation results in PLECS for $m_a = 0{,}8$ and $m_f = 40$.



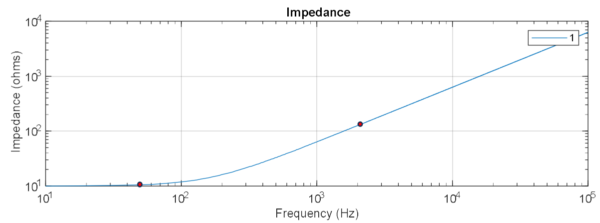


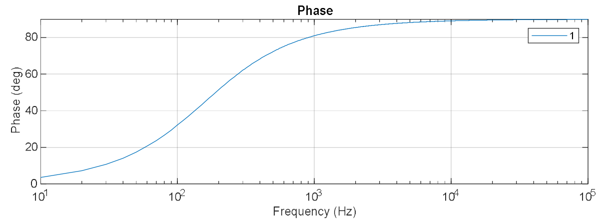
Example of half-bridge inverter $R = 10\ \Omega$, $L = 10$ mH, $V_{dc} = 50$ V $m_a = 0{,}8$ and $m_f = 40$ ($f_S = 2$ kHz).



RL load has impedance $Z(f)$

$$Z(f) = \sqrt{R^2 + (2\pi f L)^2}\, e^{j\cdot \arctan\left(\frac{2\pi f L}{R}\right)}$$





For $f = 50$ Hz $|Z| = 10.5\ \Omega$

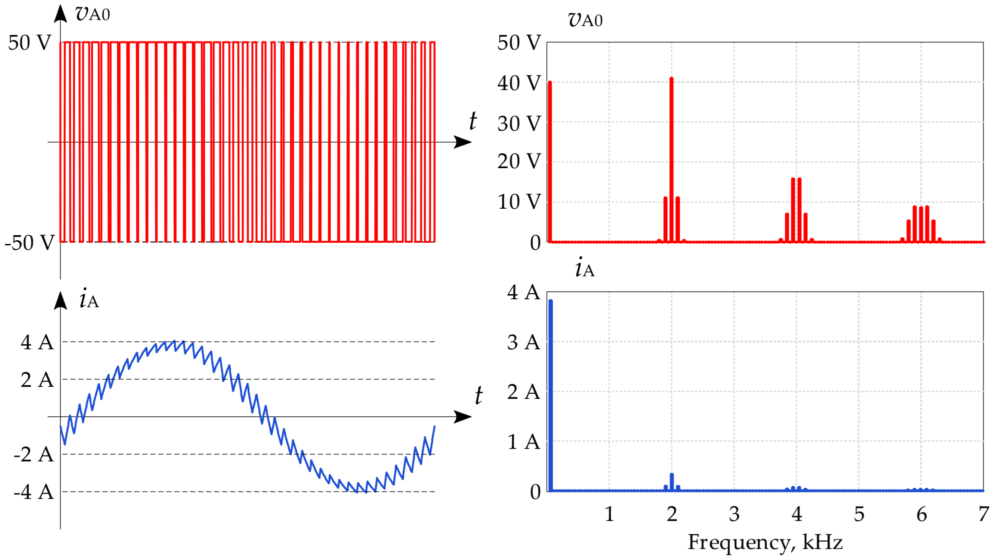
$$I_{O1} = \frac{m_a \frac{V_{dc}}{2}}{|Z|} = \frac{40\,\text{V}}{10{,}5} = 3.8\,\text{A}$$

For $f = 2$ kHz $|Z| = 126\ \Omega$

$$I_{Omf} = \frac{0{,}82 \cdot 50\,\text{V}}{126\,\Omega} = 0.32\,\text{A}$$

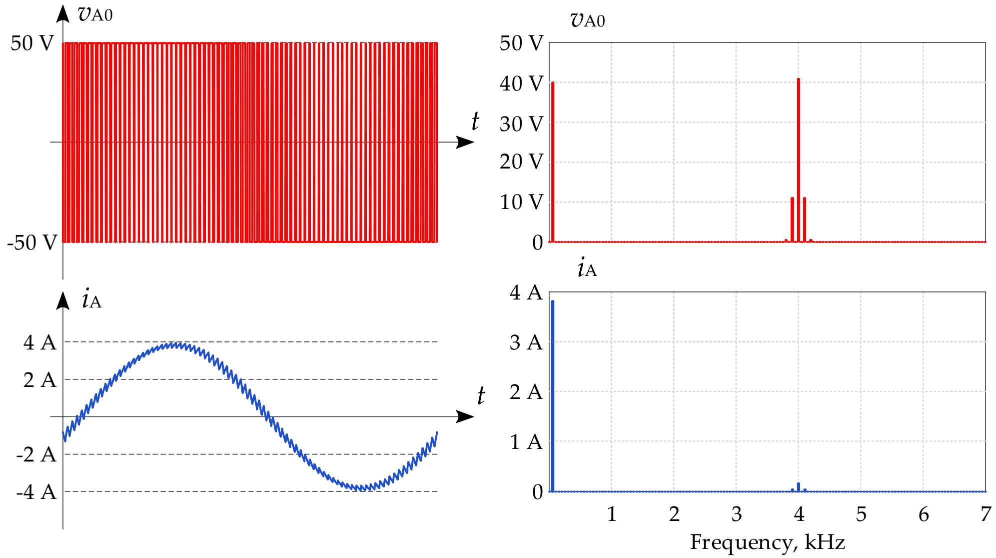
Pulse width modulation – harmonic spectrum

Simulation results in PLECS for m_a = 0,8 and m_f = 40



h	1	38	40	42	77	79	81	83
V_{A0h}, V	40.0	11.0	41.0	11.0	7.0	15.7	15.7	7.0
$\|Z\|$, Ω	10.48	119.8	126.1	132.3	242.1	248.4	254.7	260.9
I_{ah}, A	3.82	0.092	0.324	0.083	0.029	0.063	0.062	0.026

Simulation results in PLECS for m_a = 0,8 and m_f = 80

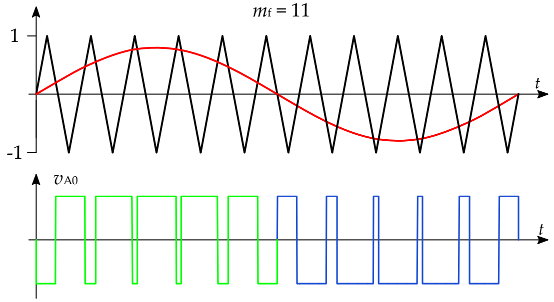


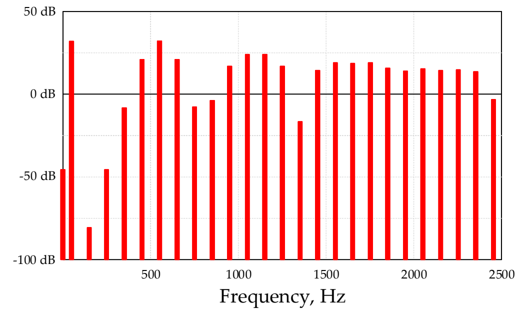
$$I_{ah} = \frac{V_{A0h}}{|Z_h|}$$

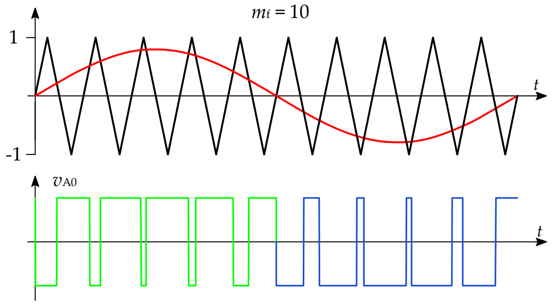
h	1	78	80	82
V_{A0h}, V	40.0	11.0	41.0	11.0
$\|Z_h\|$, Ω	10.48	245.2	251.5	257.8
I_{ah}, A	3.82	0.045	0.163	0.043

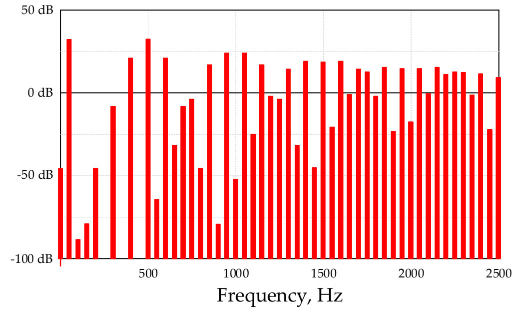
Pulse width modulation – Are there any limitations on m_f?

1. Can the frequency index m_f be even?





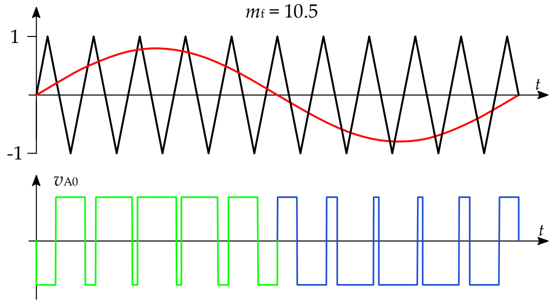


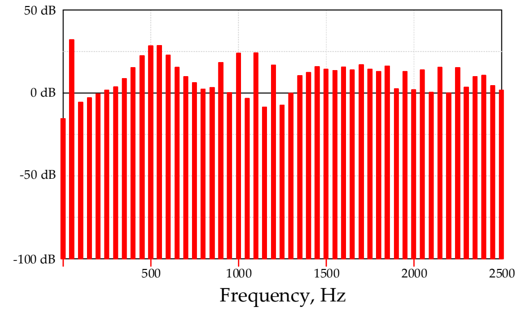


For even values of m_f, even harmonics appear in the spectrum, but their amplitudes are very small.

In both cases $THD_U = 145.768\%$.

2. Can the frequency index m_f be non-integer?





In this case, the voltage spectrum contains all **even** and **odd** harmonics, and a dc component also appears.

The value of the THD coefficient was determined $THD_U = 143.94\%$!!!

In this case, spectral leakage occurs (the component of frequency 10.5 f_m leaks into the 10th and 11th harmonics).

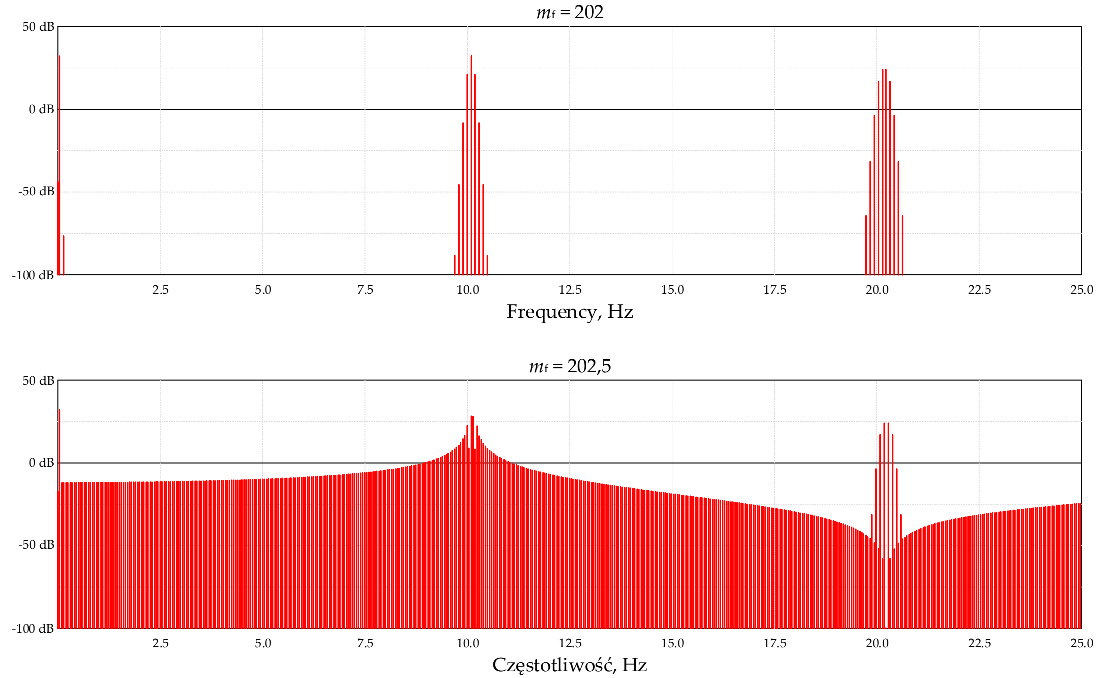
Pulse width modulation – large values of m_f

For large values of m_f, even harmonics appear in the voltage spectra, or leakage phenomena occur.

The influence of these additional components on the THD value is negligible and is not a reason to avoid using even frequency indices or those that are non-integer.

In high-power converters, where the switching frequency is low, the discussed issues are important.

In grid-connected converters, where the fundamental harmonic depends on the variable grid frequency, it is not possible to maintain an integer ratio m_f.

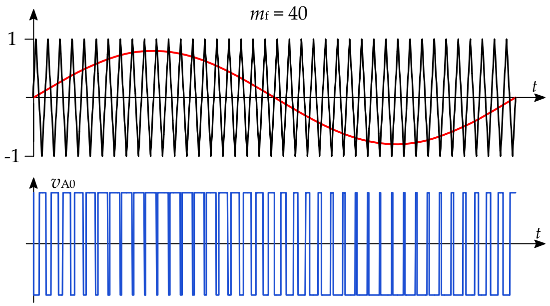


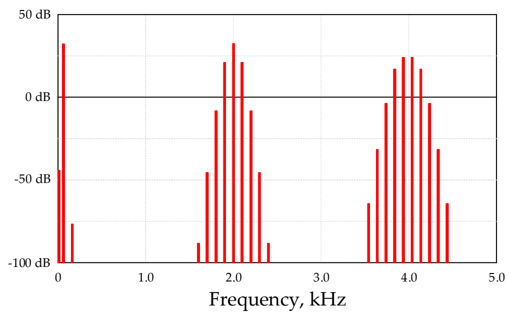
For three-phase inverters, it is recommended that m_f be a multiple of 3 e.g., 39, 201.

Pulse width modulation – carrier waveform influence

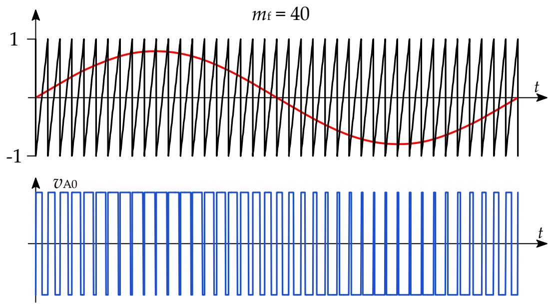
The carrier signal analysed so far had a triangular shape, which is the recommended carrier waveform for most PWM modulation schemes. Another possible carrier is the sawtooth waveform, commonly used in DC-DC converters.

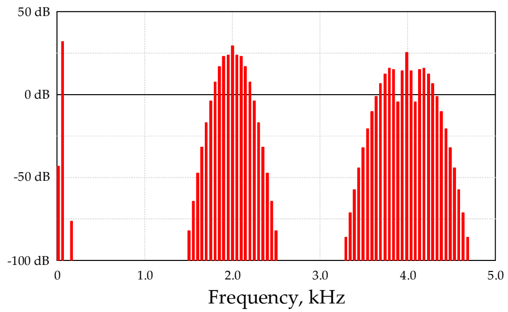
The harmonic spectrum of the output voltage when using PWM with a triangular carrier for $m_f = 40$ ($m_a = 0{,}8$) and THD is 145,76%.





Harmonic spectrum of the output voltage when using PWM with a sawtooth carrier for $m_f = 40$ i $m_a = 0{,}8$. THD is the same as for the triangular carrier, 145,76%.





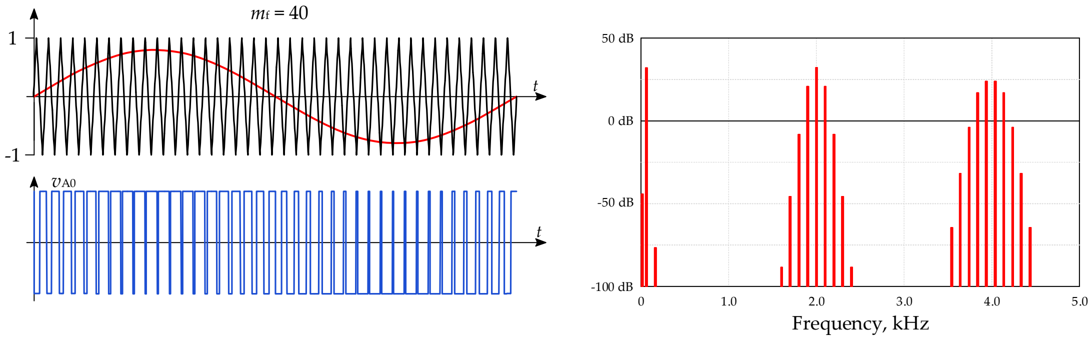
In the sidebands of harmonics associated with the carrier frequency, both even and odd harmonics appear. However, the presence of these harmonics does not significantly affect the value of the voltage THD coefficient.

Pulse width modulation – regular sampling method

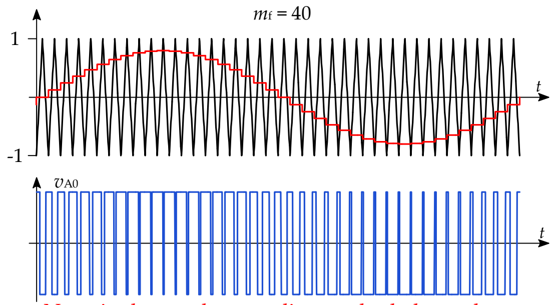
In modern microprocessor-based systems, PWM signals are generated using dedicated modules in which the value of the modulating signal is cyclically updated.

The method in which the modulating signal is updated once or twice per switching period is called the **regular sampling method**. The methods discussed earlier are natural sampling methods

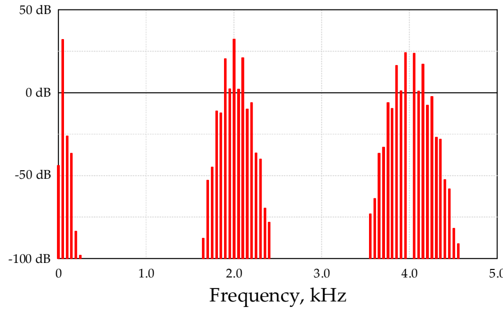
Natural sampling method



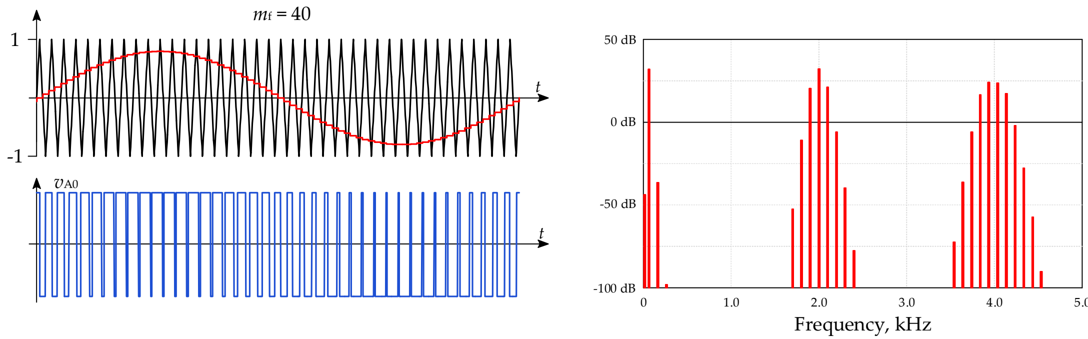
Regular sampling method with single update at S_N = -1



Note: in the regular sampling method, the update should occur at −1 or +1 of the carrier signal



Regular sampling method with double update at S_N = -1 and S_N = 1



In all presented cases THD_U is the same and equals 147%.

Comparison of harmonic spectra

Normalized voltage harmonics and THD for $m_f = 39$ and $R = 10\ \Omega$, $L = 10$ mH, $V_{dc} = 50$ V.

$V_{A0h}/(V_{dc}/2)$		Natural method		Regular method		
		S_N triangular	S_N sawtooth	S_N triangular	S_N sawtooth	S_N tri DU
$m_f = 39$ (odd number)						
m_f	odd	0.8181	0.6016	0.8181	0.6016	0.8181
$m_f \pm 1$	even	-	0.3144	0.0260	0.2960	-
$m_f \pm 2$	odd	0.2198	0.2851	0.2271	0.2808	0.2278
$m_f \pm 3$	even	-	0.1395	0.0066	0.1509	-
$m_f \pm 4$	odd	0.0076	0.0478	0.0099	0.0596	0.0101
$2m_f$	even	-	0.3721	-	0.3721	-
$2m_f \pm 1$	odd	0.3143	0.1052	0.3049	0.1057	0.3052
$2m_f \pm 2$	even	-	0.0119	0.0228	0.0021	-
$2m_f \pm 3$	odd	0.1395	0.1146	0.1443	0.0988	0.1454
$2m_f \pm 4$	even	-	0.1250	0.0086	0.1210	-
THD_U		145.77	145.77	145.97	145.88	145.80
THD_I		9.8036	9.8144	9.8145	10.2274	9.8007

Normalized voltage harmonics and THD for $m_f = 40$ and $R = 10\ \Omega$, $L = 10$ mH, $V_{dc} = 50$ V.

$V_{A0h}/(V_{dc}/2)$		Natural method		Regular method		
		S_N triangular	S_N sawtooth	S_N triangular	S_N sawtooth	S_N tri DU
$m_f = 40$ (even number)						
m_f	even	0.8181	0.6016	0.8181	0.6016	0.8181
$m_f \pm 1$	odd	-	0.3144	0.0253	0.2965	-
$m_f \pm 2$	even	0.2198	0.2851	0.2269	0.2809	0.2276
$m_f \pm 3$	odd	-	0.1395	0.0064	0.1506	-
$m_f \pm 4$	even	0.0076	0.0478	0.0099	0.0593	0.0100
$2m_f$	odd	-	0.3721	-	0.3721	-
$2m_f \pm 1$	even	0.3143	0.1052	0.3052	0.1057	0.3054
$2m_f \pm 2$	odd	-	0.0119	0.0222	0.0017	-
$2m_f \pm 3$	even	0.1395	0.1146	0.1444	0.0992	0.1452
$2m_f \pm 4$	odd	-	0.1250	0.0086	0.1211	-
THD_U		145.77	145.77	145.96	145.87	145.79
THD_I		9.5598	9.5697	9.5699	9.9721	9.5571

Analytical form of the harmonic spectrum for the PWM method

Output voltage waveform in the PWM method for a triangular carrier signal.

J_0 is the Bessel function of the first kind and 0 order

$$v_{A0}(t) = \frac{V_{dc}}{2} m_a \cos(\omega_o t + \theta_o) + \frac{2V_{dc}}{\pi} \sum_{m=1}^{\infty} \frac{1}{m} J_0\left(m\frac{\pi}{2} m_a\right) \sin\left(m\frac{\pi}{2}\right) \cos\left(m[\omega_c t + \theta_c]\right) +$$

$$+\frac{2V_{dc}}{\pi} \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ (n\neq 0)}}^{\infty} \frac{1}{m} J_n\left(m\frac{\pi}{2} m_a\right) \sin\left([m+n]\frac{\pi}{2}\right) \times \cos\left(m[\omega_c t + \theta_c] + n[\omega_o t + \theta_o]\right)$$

Output voltage waveform in the PWM method for a sawtooth carrier signal.

$$v_{A0}(t) = \frac{V_{dc}}{2} m_a \cos(\omega_o t + \theta_o) + \frac{V_{dc}}{\pi} \sum_{m=1}^{\infty} \frac{1}{m} \left[\cos(m\pi) - J_0(m\pi m_a)\right] \sin\left(m[\omega_c t + \theta_c]\right) +$$

$$+\frac{V_{dc}}{\pi} \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ (n\neq 0)}}^{\infty} \frac{1}{m} J_n(m\pi m_a) \begin{bmatrix} \sin\left(n\frac{\pi}{2}\right) \times \cos\left(m[\omega_c t + \theta_c] + n[\omega_o t + \theta_o]\right) \\ -\cos\left(n\frac{\pi}{2}\right) \times \sin\left(m[\omega_c t + \theta_c] + n[\omega_o t + \theta_o]\right) \end{bmatrix}$$

Comments on PWM harmonic spectrum comparison

1. The most favorable harmonic spectrum is observed for the natural sampling method with a triangular carrier signal and for the regular method with a triangular carrier signal with double updating.
2. The observed absence of harmonics in the spectrum when using these methods is independent of whether m_f is even or odd. However, when m_f is odd, the harmonics that are absent in the spectrum are even harmonics. This is a beneficial feature, and it is recommended that m_f be odd.
3. When using a sawtooth carrier signal, additional harmonics appear in the voltage spectrum with a slight reduction the value of the m_f harmonic.
4. The voltage THD value is very similar in most cases.
5. The highest current THD value is observed in the regular method with a sawtooth carrier signal. This method is not recommended for modulating inverter signals.
6. It is important to remember that in the regular sampling method, the modulating signal is updated at the minimum carrier signal value (-1 or 0). For a different carrier signal value, the voltage harmonic spectrum may change.
7. The double update method is the most recommended PWM method.

Conclusions

1. Different variants of two-level converter were shown.
2. The role of application in the analysis of the midpoint of a dc circuit was explained.
3. The different types of voltages occurring in an inverter are presented, and the common mode voltage in three-phase inverters was explained.
4. The basics of the PWM modulation method were explained.
5. The influence of modulation parameters (modulation index, frequency index, fundamental harmonic frequency) on output voltage and current waveforms was discussed.
6.
7. The harmonic spectrum of the voltage of a half-bridge inverter was presented, with harmonics divided into harmonics surrounding the fundamental harmonic, the carrier signal harmonic, and their sidebands.
8. The influence of harmonics on the output current harmonics was explained.
9. The effect of different values of the mf factor (even, odd, and fractional) was analyzed.
10. Spectra for triangular and sawtooth carrier signals were presented.
11. Spectra for the regular sampling method were presented.

Thank you for your attention

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